Review Questions.

12. Determine if the following integrals converge or diverge.

(a) $$\int_2^{\infty} \frac{x^{10} + e^x + 30\cos(x) - 1}{x^3 e^x + 2x^{75}}\,dx$$

(b) $$\int_2^{\infty} \frac{x^{10} + e^x + 30\cos(x) - 1}{x\left(e^x + 2x^{75}\right)}\,dx$$

(c) $$\int_2^{3} \frac{2b^2 - 7b^5 + 4b\left(5^{b-1}\right)}{\left(2b^2 - b - 15\right) b^{3/2}}\,db$$

(d) $$\int_0^{2} \frac{2b^2 - 7b^5 + 4b\left(5^{b-1}\right)}{\left(2b^2 - b - 15\right) b^{3/2}}\,db \quad 0$$

near $b=3$: $\approx \dfrac{2\cdot 3^2 - 7\cdot 3^5 + 4\cdot 3\, 5^2}{11\cdot(b-3)\, 3^{3/2}}$

$(2b+5)(b-3)$

near $b=0$: $\approx \dfrac{4b5^{b-1}}{-15 b^{3/2}} \approx \dfrac{4b}{-75\, b^{3/2}} \approx \dfrac{4}{-75}\dfrac{1}{b^{1/2}}$

Two things to keep in mind:

1. Where is the problem where the area is infinite?
   - a # where denom = 0
   - ∞, $\pm$
2. What parts of numerator/denominator are the dominant terms, and what is insignificant?

Example Does this integral converge or diverge?

$$\int_{5000}^{\infty} \frac{x^2 - 2e^x + 7e^{-x} + x^{10000}\cos(3x)}{5e^x + 2e^{-x} + x^2e^x - \sin(x)}\,dx$$

denominator $\neq 0$, because $5e^x + 2e^{-x} + x^2e^x$ is much bigger than 1, so it's always positive.

We should be examining the terms for x very large & positive.

For very large x, fcn $\approx \frac{-2e^x}{x^2e^x} = \frac{-2}{x^2}$

Since $\int_{5000}^{\infty} \frac{2}{x^2}dx$ converges, we expect ours to converge.

$$\frac{x^2 - 2e^x + 7e^{-x} + x^{10000}\cos(3x)}{5e^x + 2e^{-x} + x^2e^x - \sin(x)}$$

$$= \frac{\frac{x^2}{e^x} - 2 + \frac{7}{e^{2x}} + \frac{x^{10000}}{e^x}\cos(3x)}{5 + \frac{2}{e^{2x}} + x^2 - \frac{\sin(x)}{e^x}}$$

For some large x (say $x > M$),

$$\left| \underset{\searrow 0}{\frac{x^2}{e^x}} + \underset{\searrow 0}{\frac{7}{e^{2x}}} + \underset{\searrow 0}{\frac{x^{10000}}{e^x}}\cos(3x) \right| < 1$$

$|\text{blah}| < 1$

$-1 < \text{blah} < 1$

$-2 \quad -2 \quad -2$

for $x > M$,

blah -2

$$\left| \underbrace{\frac{x^2}{e^x} - 2 + \frac{7}{e^{2x}} + \frac{x^{10000}}{e^x}\cos(3x)}_{\text{between } -3 \text{ \& } -1.} \right| < 3$$

Also, for $x > M$,

$$5 + \underset{\to 0}{\frac{2}{e^{2x}}} + x^2 - \frac{\sin(x)}{e^x} > x^2 + 4 > x^2$$

My integral $= \underbrace{\int_{5000}^{M}}_{\text{finite}} + \underbrace{\int_{M}^{\infty}}$

$$0 \le \left| \int_M^\infty \sim \right| \le$$

$$0 \le \int_M^\infty \frac{3}{x^2}\,dx \leftarrow \text{finite.}$$

Thus, the whole integral is finite.

ⓒ $\left| f(n) \right| \ge \left| \frac{1}{11 \cdot 3^{3/2}(b-3)} \right|$

$\therefore$ since $\int_2^3 \frac{1}{b-3}\,db$ diverges,

so does $\int_2^\infty (f(n))$.

If $|f(x)| > |g(x)|$

Then $\int_a^b |f(x)|\,dx \ge \int_a^b |g(x)|\,dx$